

Homework 3. Numeric integration of a linear triangular element

Yuri Fialko, Feb. 11, 2008 (due Feb. 20)

1) Generation of a simple mesh using Matlab's Delaunay triangulation: populate a circle with a number of points such that x and y are the respective coordinate vectors. Create a Delaunay connectivity using

```
tri=delaunay(x,y);
```

Plot the mesh using `trimesh(tri,x,y)`;

2) Write an M-file `I=triang_int(f,v1,v2,v3)` which inputs are a function $f(x,y)$ (written as a symbolic expression), and three 2×1 matrices $v1$, $v2$, and $v3$ representing vertices (in any order) of a triangle in the xy plane. If we denote this triangle by T , the output I will be the numerical integral $\int \int_T f(x,y) dx dy$.

Hint. Matlab function `dblquad` allows one to compute double integrals over a rectangle. To use it over regions other than rectangles, the following identity may be useful,

$$\int \int_T f(x,y) dx dy = \int_{x_{min}}^{x_{max}} \int_{y_{low}(x)}^{y_{top}(x)} f(x,y) dy dx = \int_{x_{min}}^{x_{max}} \int_0^1 f(x, y_{low}(x) + u(y_{top}(x) - y_{low}(x))) [y_{top}(x) - y_{low}(x)] du dx,$$

where x_{min} and x_{max} are the maximum and minimum abscissae of a surface element T . Note that this expression applies not only to a triangle, but to any region in the plane bounded below by the curve $y_{low}(x)$ and above by the curve $y_{top}(x)$ (both curves must be defined over the range $[x_{min} \ x_{max}]$). An attached Matlab script gives an example of use of `dblquad` for computing double integrals using the above procedure.

```
function nint= quad2d(fun,xmin,xmax,ylow,ytop)
%numerically computes a double integral of a function 'fun' on a region
%over the interval minx<x<maxx and between the functions of x: ylow<y<ytop.
% INPUTS: fun = a function of the symbolic variables x and y
% minx = minimum x-value for region
% maxx = maximum x-value for region
% ylow = function of symbolic variable for lower boundary of region
% ytop = function of symbolic variable for upper boundary of region
% OUTPUT: nint = numerical approximation of integral using the integrator
% 'dblquad' in conjunction with the default settings.
% x and y should be declared symbolic variables before this M-file is used.
syms u x y
ynew=ylow+u*(ytop-ylow);
funprep=subs(fun,y,ynew)*(ytop-ylow);
funnew=vectorize(inline([char(funprep),'*ones(size(u))'],'u','x'));
nint = dblquad(funnew,0,1,xmin,xmax);
```