1) Derive a second-order accurate finite difference equation for a first derivative \( du/dx \) that uses a “one-sided” stencil - that is, values \( u_{i-2}, u_{i-1}, u_i \) (or \( u_{i}, u_{i+1}, u_{i+2} \)) to evaluate the derivative of \( u \) at point \( x_i \).

2) Determine the order of accuracy of the following FD equation to the PDE
\[
\frac{du}{dt} + vu_x = 0, \\
\tag{1}
\]
\[
u_{i,j+1} = u_{i,j-1} - \frac{vu}{\Delta x}(u_{i+1,j} - u_{i-1,j})
\]
where \( v = \text{const} \), and indexes \( i \) and \( j \) correspond to discrete values of \( u(x_i,t_j) \).

3) Implement a finite difference solution to the flux-conservative initial value problem
\[
\frac{du}{dt} + vu_x = 0, \\
\tag{2}
\]
on an interval \( x=[0 1] \), subject to the initial condition \( u(x,0) = \sin(4\pi x) \) and boundary condition \( u(0,t) = 0 \). Assume \( v = 1 \). Use the first-order FD approximation for the time derivative \( \frac{du}{dt} \), and the following FD approximations for \( u_x \): (i) first-order upwind scheme; (ii) second-order centered scheme; (iii) second-order centered scheme with the Lax substitution \( u_{i,j} \rightarrow \frac{1}{2}(u_{i+1,j}+u_{i-1,j}) \). Choose \( \Delta x \) to ensure an adequate spatial resolution of your solution (a simple visual inspection will suffice for now), and explore how your FD solutions depend on a time step \( \Delta t \).