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• Conceived in the 1970’s, first satellites launched 1978, became “operational” in 1994.

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- Also realized (in academia) that GPS could be used differently, for really precise positioning (“geodesy”).
- As a result of this history, GPS is not really designed for geodesy, and some of “GPS theory” and processing involves getting around this problem.
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Why use GPS? “It’s where the money is”
Important Dates

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• 1994 A/S turned on.

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• 2000 SA turned off—much enhanced civil navigation: a GPS in every cell phone.
GPS Orbits

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Note that satellites move in “inertial space”, with the largest acceleration not from gravity being about $10^{-8} \, g$. This makes them a (nearly) perfect “outside the Earth” reference frame over short times (days).

Orbit determination is an important part of GPS geodesy, but we will ignore it (treat the satellite positions as given). This is more reasonable now than it used to be.
Note that satellites never cross the pole, so at the poles are never at the zenith: a “hole” in the sky distribution.
Where the Satellites Are (Looking Down)

SV Ground Tracks on 2007, January 17, 0200 to 0300

Green is visible from La Jolla (at least 10° elevation)

Lines show the movement over an hour.
Where the Satellites Are (Looking Up)

SV Sky Tracks on 2007, January 17, at SIO3

Viewed from SIO3 (near the Aquarium). Yellow is sky tracks over a day (notice the hole to the N), green shows the movement over an hour.
Satellites Over a Day

Left plot is satellite elevation, for several satellites. “PRN” is the unique code each satellite broadcasts (since they all transmit on the same frequency).

Right plot is distance to satellites.

Satellite-station velocities range from 0 to 900 m/s.
Satellite Availability at SIO

Top plot shows how many satellites are visible, as a function of the **elevation cutoff** (but no obstructions). The fewer, the worse are any estimates.
GPS for Navigation – Oversimplified

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The distance between the two is \( k^d = c(k^t - i^t) \) where \( c \) is the speed of light (299 792 458 m s\(^{-2}\)).
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  - Not all the radio waves travel directly from the satellite to the receiver.
Solving the Clock problem

Figure shows a GPS-like system in a flat and 2-D world, with three satellites. Given two pseudoranges, we have to be at one of two points where the circles intersect (remember, we assume we know the satellite locations).
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But with *three* satellites, the wrong time will (usually) not give us a position—we can imagine adjusting the receiver time until we do, at which point we know where we are, and what time it is (from the GPS clocks).
Differencing

More formally, we can create combinations of observables (GPS processing is about this, a lot) that remove the effects of clock errors.

Consider: $i_k d - j_k d = c(i_k t - i t) - c(j_k t - j t)$ and if the signals are received at the same time, the combination $i_k d - j_k d = c(j t - i t)$ is independent of the receiver clock and depends only on the difference in the distance between satellites; geometrically, a given distance difference gives a hyperbola that we must be on.
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For geodetic positioning, need to look at GPS signal in more detail, and consider multiple receivers.
What do the Satellites Transmit? (I)

All the radio signals are in L-band, frequencies about 1.5 GHz, wavelengths about 20 cm. More precisely, there is

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- L5: Frequency 1176.45 MHz, or 254.8 mm (not yet implemented).
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  - Timing information.
  - And lots more!
Carrier Modulation

How the GPS Signal is Modulated

GPS Signal—Cartoon Version

Actual wavelengths:
0.2 m for carrier, 30 m for P, 300 m for C/A

Actual modulation is done (as at top) by changing the phase of the carrier. Bottom plot is a cartoon of how two codes amplitude modulate carrier.
Signal Wavelengths

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To do geodesy (mm accuracy) we cannot use the codes (fine for navigation), but must use the carrier—ideally, after demodulating (which means we want to know the code).
Basic Observable: Carrier-Beat Phase

What we actually observe is the phase of the carrier as a function of time; say, at a specified time the phase is zero, and later is 90°, we know that the distance has changed by $\lambda/4$; combining such phase changes with knowing the satellite positions can gives our location.
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Actually use the carrier-beat phase: the phase of \( \exp(2\pi i [j f - k f] t) \) which is the difference of the carrier from satellite \( j \) and the oscillator driving the local (receiver \( k \)) clock. This does not vary at 1.5 GHz, which makes it much more manageable.
Propagation Delay

The velocity $c$ is not constant along the path, because of:

- The ionosphere, from 400 to 60 km, which contains charged particles.
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Tropospheric delays can be up to 2.5 m, though a simple model of the atmosphere will express them pretty well – but not well enough for geodesy.
Propagation Delay: Solutions

Ionospheric delay depends on frequency, so we can combine two frequencies to get an ionosphere-free observable.
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Ionospheric delay depends on frequency, so we can combine two frequencies to get an ionosphere-free observable. Because the atmosphere is well-mixed, we can model the dry delay using a hydrostatic atmosphere. Local meteorological data (pressure and temperature) can be used but are not much of an improvement.
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The **wet delay** cannot be modeled *a priori*, and cannot easily be measured independently, so it is usually estimated using the GPS data. More precisely, we assume the delay to have the form $Z(t)M(\theta)$ where

$Z(t)$ is the **zenith delay**, which is allowed to vary with time.

$M(\theta)$ is a **mapping function** of the elevation angle (only), decided on in advance from atmospheric models/data.
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For very small networks, the paths to the receivers, and hence the propagation delays, are nearly the same, so none of these corrections are needed (and if they are included the solution will be worse).
Propagation Delay

Delays in a Standard Atmosphere

Mapping Functions
Local Effects I: Antenna Phase Delay

The ideal antenna would respond only to signal above the horizon, and not introduce any time delay: neither is realistic. In fact, if the ideal signal were $U_0 e^{2 \pi i ft}$ the actual one will have two additional terms.
Local Effects I: Antenna Phase Delay

The ideal antenna would respond only to signal above the horizon, and not introduce any time delay: neither is realistic. In fact, if the ideal signal were $U_0 e^{2\pi ft}$ the actual one will have two additional terms.

First, we will have $U_0 e^{2\pi ft} [e^{i\phi_A(\theta_0, \beta_0)}]$ where $\phi_A$ is the phase shift introduced by the antenna itself, as a function of the elevation angle $\theta_0$ and azimuth $\beta_0$ of the incoming signal; this shift includes any offset of the antenna “phase center” from the reference point on the antenna.

In general, this will be reduced if the same antenna types are used, or the differences modeled.
Local Effects II: Multipath

In addition, we will have a term $U_0 e^{2\pi ift} \left[ \int_{\Omega^-} A(\theta, \phi) R(\theta, \phi) e^{i\phi R(\theta, \phi)} d\theta d\phi \right]$ The integral term is meant to include all the “multipath” contributions, and so is an integral over $\Omega^-$, which denotes the unit sphere excluding the direction of the direct wave. This includes
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- Scattering from the antenna support.
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This cannot be modeled, and presents a noise source that limits the precision of measurements made over short times.