GEOPHYSICAL RESEARCH LETTERS, VOL. 22, NO. 18, PAGES 2541-2544, SEPTEMBER 15, 1995

# Fracture criteria at the tip of fluid-driven cracks in the earth

Yakov M. Khazan

Department of Tectonophysics, Institute of Geophysics, Kiev, Ukraine

Yuri A. Fialko

Department of Geological and Geophysical Sciences, Princeton University, Princeton, New Jersey

**Abstract.** The effect of high confining pressure on fluid-filled crack growth is considered. Exact solutions are given for a two-dimensional horizontal crack in an infinite elastic body using the approximation of Dugdale-Barenblatt (DB) model. It is shown that for equilibrium cracks (i.e. for cracks on the verge of propagation) the large-scale crack characteristics, such as fluid overpressure, apparent fracture toughness, maximum opening of the crack and crack volume, grow with increase of confining pressure. These effects result from a pressure induced fracture resistance (PIFR). If basic parameters of the DB model (tensile strength and critical crack opening displacement) are independent of confining pressure then PIFR dominates over intrinsic rock strength starting from quite shallow depth (tens to hundreds of meters).

## Introduction

Experimental investigations demonstrate that tensile fracture resistance, which is commonly characterized by fracture toughness $K_{Ic}$ [*Atkinson and Meredith,* 1987], grows with the increase of confining pressure [*Perkins and Krech,* 1966; *Abou-Sayed,* 1977; *Schmidt and Huddle,* 1977; *Thallak et.al.*, 1993]. This increase in resistance may be partially related to the effect of confining pressure on fracture mechanism in the process zone (the region of inelastic deformation where fracture actually takes place). At the same time, it is obvious that confining pressure decreases tensile stresses in the crack tip region, leading to the apparent increase of strength. This pressure-induced fracture resistance is quite significant. As we will show below, the level of fluid pressure when a fracture is on the verge of propagating is determined by confining (lithostatic) pressure rather than by intrinsic rock strength starting from quite shallow depth.

LEFM (Linear Elastic Fracture Mechanics) is valid, strictly speaking, only in the case of ideally brittle fracture. In reality fracture is always preceded by inelastic deformation (microcracking or plastic flow) near the crack tip. Classical LEFM is applicable if the inelastic zone is small compared with other characteristic dimensions of the problem [*Irwin*, 1957]. However, experimental data for metals as well as for rocks [*Labuz et.al.*, 1987; *Swanson,* 1987] show that the process zone dimensions commonly are not small compared to typical laboratory specimen sizes. Recently *Rubin* [1993] pointed out that at high confining pressures there is no $K$-dominant region surrounding the process zone, which means that classical LEFM approach cannot be used. There exist, nonetheless, many approaches that allow one to get rid of nonlinearity or to put it in the boundary conditions. One such an approach is the Dugdale-Barenblatt (DB) model [*Dugdale*, 1960; *Barenblatt*, 1962], which is also referred to as a tension-softening model [*Hashida et.al.*, 1993].

The DB model holds that a thin plastic zone forms in the crack plane ahead of the crack tip. Cohesive forces $\sigma_T(\delta)$, which generally depend on crack wall separation $\delta$, act within this zone perpendicular to the crack plane. Fracture is usually assumed to take place when critical opening displacement $\delta_c$ is exceeded. Cohesive forces can be related to surface energy $\gamma$ or fracture energy $G_c$ calculated over the unit thickness of a specimen,

$$2\gamma = G_c = \int_0^{\delta_c} \sigma_T(\delta)\, d\delta, \qquad (1)$$

or to the invariant $J$-integral [*Rice,* 1968]. Existence of thin in-plane process zones is confirmed experimentally. They were observed in metals by *Dugdale* [1960] and in rocks by *Swanson* [1987]. Measurements of critical opening displacement $\delta_c$ for metals as well as for rocks gave rise to the values of $\delta_c \sim 0.01\ mm$, large compared to a crystal lattice constant, which suggests that the fracture mechanism is plastic rather than brittle.

Although the nature of cohesive forces is not fully understood, the DB model may be successfully employed to interpret rock fracture experiments [*Hashida et.al.*, 1993]. There exist numerous solutions for tensile cracks at zero confining pressure [see e.g. *Rice,* 1968]. At the same time, fluid-driven cracks under high confining pressures are of considerable interest from the point of view of geophysical applications. Below we present exact solutions for the DB model for cracks in an infinite body assuming ideal plasticity in the process zone (i.e. $\sigma_T(\delta) = const$).

## Mathematical formulation

Let us consider a two-dimensional crack having length $2a$ in an infinite elastic body subjected to a confining pressure $p_\infty$ (Figure 1). Internal pressure opening the crack is applied to its surface on the interval $|x| < a - \Delta_T$; cohesive stresses $\sigma(x) = \sigma_T$ act on the interval $a - \Delta_T < |x| < a$. The crack is assumed to be in equilibrium (i. e. on the verge of propagation); in terms of the DB model this means the crack wall separation at the base of the cohesive zone ($|x| = a - \Delta_T$) equals the critical opening $\delta_c$. Let us also suppose that unpenetrated regions (tip cavities) having length $\Delta_S$ and internal pressure $p_S$ that is less than fluid pressure $p$ exist near crack tips [*Rubin*, 1993]. Thus, the boundary conditions on the crack walls may be written as



Paper number 95GL02547
0094-8534/95/95GL-02547$03.00

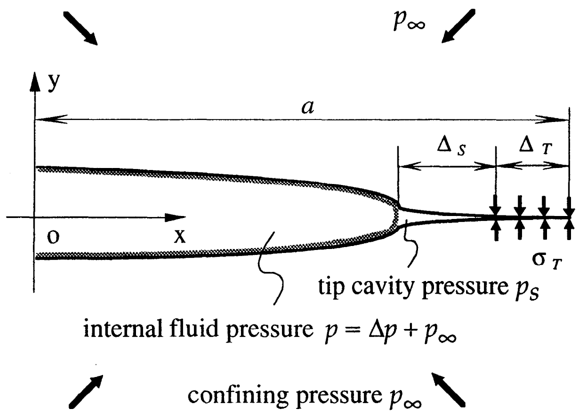


**Figure 1.** Schematic view of a plain strain problem for a two-dimensional crack in an infinite elastic body subjected to internal fluid pressure $p$ and hydrostatic confining pressure $p_\infty$. Half-length of the crack is $a$, $\Delta_S$ is unwetted region ahead of the fluid front, having pressure $p_S$, and $\Delta_T$ is a cohesive zone; at the base of the cohesive zone ($x = a - \Delta_T$) crack opening equals critical opening displacement $\delta_c$ (not shown).

follows:

$$\sigma(x) = \begin{cases} -p, & 0 < |x| < a - (\Delta_T + \Delta_S) \\ -p_S, & a - (\Delta_T + \Delta_S) < |x| < a - \Delta_T \\ \sigma_T, & a - \Delta_T < |x| < a. \end{cases} \qquad (2)$$

Since the function $\sigma(x)$ was chosen to be symmetric with respect to the origin (center of the crack), results obtained below correspond to horizontal cracks or to the cracks in which changes in hydrostatic pressure are small compared to fluid pressure $p$.

In the case of plain strain deformation symmetric with respect to the $x$ axis, the solutions to the equations of theory of elasticity can be found from Kolosov-Muskhelishvili formulae [see e.g. *Muskhelishvili*, 1954; *Tada et.al.*, 1973]:

$$\sigma_{yy} = \mathrm{Re}\ \varphi' + y\ \mathrm{Im}\ \varphi'' \qquad (3)$$

$$2\mu U_y = 2\ (1-\nu)\ \mathrm{Im}\ \varphi - y\ \mathrm{Re}\ \varphi', \qquad (4)$$

where $\mu$ stands for the shear modulus, $\nu$ is Poisson's ratio and $\varphi(z)$ is an analytical function of the complex variable $z = x + iy$. Displacements $U_y$ equal zero on the real axis outside the crack interval (i.e. $\mathrm{Im}\,\varphi = 0$ for $y = 0$, $|x| > a$). Differentiating the last condition and keeping in mind that stresses should approach $p_\infty$ at infinity, we get the following boundary conditions for the function $\varphi'$:

$$\begin{aligned} &\mathrm{Re}\ \varphi'(z) = \sigma(x), \quad |x| < a \\ &\mathrm{Im}\ \varphi'(z) = 0, \quad |x| > a \\ &\varphi'(z) \to -p_\infty, \quad |z| \to \infty. \end{aligned} \qquad (5)$$

The explicit form of $\varphi'(z)$ can be found using Keldysh - Sedov formula [*Muskhelishvili*, 1953; *Lavrent'ev and Shabat*, 1958]:

$$\varphi'(z) = \frac{1}{\pi i}\sqrt{\frac{z+a}{z-a}}\int_{-a}^{a}\sigma(t)\sqrt{\frac{t-a}{t+a}}\frac{dt}{t-z} + \frac{a}{\pi}\int_{-a}^{a}\frac{\sigma(t)+p_\infty}{\sqrt{z^2-a^2}}\frac{dt}{\sqrt{a^2-t^2}} - p_\infty\sqrt{\frac{z+a}{z-a}}. \qquad (6)$$

From (3) and (6) one may find stresses in the crack plane ($y = 0$, $x = a + \xi$, $\xi > 0$):

$$\sigma_{yy}(a+\xi)\big|_{y=0} = -\frac{1}{\pi}\frac{a+\xi}{\sqrt{2a\xi+\xi^2}}\int_{-a}^{a}(\sigma(t)+p_\infty)\frac{dt}{\sqrt{a^2-t^2}} + \frac{1}{\pi}\sqrt{2a\xi+\xi^2}\int_{-a}^{a}\sigma(t)\frac{dt}{\sqrt{a^2-t^2}\,(a+\xi-t)}. \qquad (7)$$

As $\xi \to 0$, the second term in (7) goes to $\sigma_T + O(\xi^{1/2})$, i.e. remains finite, so that the condition for the absence of tip singularity may be written as:

$$\int_{-a}^{a}\frac{\sigma(t)+p_\infty}{\sqrt{a^2-t^2}}\,dt = 0, \qquad (8)$$

which leads to

$$\Delta p\ \arcsin S = (p_\infty - p_S)\ \arccos S + (\sigma_T + p_S)\ \arccos T. \qquad (9)$$

In equation (9) $\Delta p$ is fluid overpressure, or driving pressure, $\Delta p = p - p_\infty$, $S = 1 - (\Delta_T + \Delta_S)/a$ and $T = 1 - \Delta_T/a$.
For $\Delta_T$, $\Delta_S << a$,

$$\Delta p = \sqrt{\frac{8}{\pi^2 a}}\left((p_\infty - p_S)\sqrt{\Delta_T + \Delta_S} + (\sigma_T + p_S)\sqrt{\Delta_T}\right). \qquad (10)$$

The crack opening $\delta(x)$ can be found from equations (4), (6) and (8):

$$\delta(x) = \frac{2(1-\nu)}{\pi\mu}\int_{x}^{a}d\tau\sqrt{a^2-\tau^2}\int_{-a}^{a}\frac{\sigma(t)dt}{(t-\tau)\sqrt{a^2-t^2}}. \qquad (11)$$

Taking the integral, we get:

$$\delta(x) = \frac{2(1-\nu)}{\pi\mu}a\left[(p-p_S)I(X,S) + (\sigma_T + p_S)I(X,T)\right], \qquad (12)$$

where $X = x/a$ and

$$I(U,V) = (V+U)\ln\left|\frac{\sqrt{(1-U^2)(1-V^2)}+1+UV}{V+U}\right| + (V-U)\ln\left|\frac{\sqrt{(1-U^2)(1-V^2)}+1-UV}{V-U}\right|. \qquad (13)$$

The crack opening at the base of the process zone is then

$$\delta(a-\Delta_T) = \frac{2(1-\nu)}{\pi\mu}a\left[(p-p_S)\mathrm{I}(T,S) + 2(\sigma_T + p_S)T\ \ln\frac{1}{T}\right] \qquad (14)$$

and the opening at the center is

$$\delta_{\max} = \delta(0) = \frac{4(1-\nu)}{\pi\mu}a \cdot \left[(p-p_S)S\ln\frac{1+\sqrt{1-S^2}}{S} + (\sigma_T + p_S)T\ln\frac{1+\sqrt{1-T^2}}{T}\right]. \qquad (15)$$

If one specifies the DB model parameters ($\delta_c$ and $\sigma_T$) and loading configuration ($a$, $p_\infty$ and $\Delta_S$ or $\Delta_S/\Delta_T$), equations (9) and (14) provide corresponding equilibrium values of overpressure $\Delta p$ and process zone length $\Delta_T$.
For $\Delta_S = 0$ (fluid fills the entire crack)

$$\delta(a-\Delta_T)=\frac{4(1-\nu)}{\pi\mu}a\,(\sigma_T+p)\,T\ln\frac{1}{T}, \tag{16}$$

$$\delta_{\max}=\frac{4(1-\nu)}{\pi\mu}a\,(\sigma_T+p)\,T\ln\frac{1+\sqrt{1-T^2}}{T}. \tag{17}$$

For $\Delta_S=0$ and $\Delta_T<<a$,

$$\delta(a-\Delta_T)=\frac{4(1-\nu)}{\pi\mu}\Delta_T(\sigma_T+p). \tag{18}$$

For $\Delta_T, \Delta_S<<a$,

$$\delta_{\max}=\frac{4\sqrt{2a}(1-\nu)}{\pi\mu}\left((p-p_S)\sqrt{\Delta_T+\Delta_S}+(\sigma_T+p_S)\sqrt{\Delta_T}\right). \tag{19}$$

It follows from equation (10) that in the limit of large cracks ($a>>\Delta_T$) and not very small confining pressures ($p_\infty\geq\sigma_T$), excess pressure $\Delta p$ is small compared to confining pressure and, consequently, $p\approx p_\infty$. In this case equations (10) and (19) yield

$$\delta_{\max}=\frac{2(1-\nu)}{\mu}a\Delta p. \tag{20}$$

Finally, from (11) one can find the volume $V$ of an equilibrium crack, taken over the layer of unit thickness:

$$V=2\int_0^a\delta(x)\,dx$$
$$=\frac{2(1-\nu)a^2}{\mu}\left[(p-p_S)\,S\sqrt{1-S^2}+(\sigma_T+p_S)T\sqrt{1-T^2}\,\right]. \tag{21}$$

In the same limiting case when equation (20) is valid,

$$V=\frac{\pi(1-\nu)}{\mu}a^2\,\Delta p. \tag{22}$$

## Pressure Induced Fracture Resistance

The solutions above show that the large scale values that characterize an equilibrium crack (excess pressure $\Delta p$, opening at the center $\delta_{\max}$, volume $V$) grow with the confining pressure $p_\infty$ (or with depth, if cracks in the Earth are considered). It is evident that apparent fracture toughness, if one formally introduces the latter as $K_Q=\Delta p\sqrt{\pi a}$, has the same kind of pressure dependence. In the limit of a large crack ($\Delta_T<<a$) and zero tip cavity pressure equation (10) implies that

$$K_Q=\sigma_T\sqrt{\frac{8\Delta_T}{\pi}}+p_\infty\sqrt{\frac{8(\Delta_T+\Delta_S)}{\pi}}. \tag{25}$$

Here it is instructive to compare equation (25) with corresponding expression resulting from LEFM. In the approximation of LEFM stress at the crack tip $\sigma_{\text{tip}}$ may be written as

$$\sigma_{\text{tip}}=\frac{K_I}{\sqrt{2\pi r}}-p_\infty, \tag{26}$$

where $K_I$ is mode I stress intensity factor, $K_I=\Delta p\sqrt{\pi a}$, and $r$ is the tip radius. $K_I$ equals apparent fracture toughness $K_Q^{LEFM}$ when the crack is on the verge of propagation. Keeping in mind that the tip stress in this case is related to the critical stress intensity factor $K_{Ic}$, which is regarded as a (pressure dependent) rock property, $\sigma_{\text{tip}}=K_{Ic}/\sqrt{2\pi r}$, from (26) one may deduce that

$$K_Q^{LEFM}=K_{Ic}+p_\infty\sqrt{2\pi r}. \tag{27}$$

Rewriting (25) as

$$K_Q=K_{Ic}^*+p_\infty\sqrt{2\pi\xi_{eff}}, \tag{28}$$

we see that $K_{Ic}^*=\sigma_T\sqrt{8\Delta_T/\pi}$ plays the role of "intrinsic" fracture toughness and the value $\xi_{eff}=4(\Delta_T+\Delta_S)/\pi^2$ should be called effective tip radius. Obviously, the latter has nothing to do with the actual tip radius, strongly depends on loading geometry and may in fact significantly exceed the process zone size (which is of the order of $\Delta_T$). Note that $K_{Ic}^*$ is also dependent on the loading configuration through $\Delta_T=\Delta_T(p_\infty,\Delta_S)$ [*Rubin*, 1993]. Thus, it should be concluded that at high confining pressures there is no definite set of rock properties that can be effectively used as fracture criteria in the frame of LEFM.

Growth of equilibrium overpressure $\Delta p$ (or apparent fracture toughness $K_Q$) with confining pressure reflects an effective increase of rock strength with depth. The component of fracture toughness that scales with confining pressure $p_\infty$ may be called pressure induced fracture resistance (PIFR). Existence of PIFR qualitatively explains the rock strength increase with confining pressure observed in many experiments [*Schmidt and Huddle,* 1977; *Atkinson and Meredith,* 1987; *Thallak et.al.*, 1993], although available data are not sufficient to evaluate the effect of (possible) pressure dependence of $\sigma_T$ and $\delta_c$ on experimental results. Note that the effect of confining pressure on apparent fracture toughness $K_Q$ depends not only on the pressure magnitude, but also on the effective tip radius $\xi_{eff}$ (equation (23)). This may be the reason why the slopes of the $K_Q(p_\infty)$ curves for the same material at different experimental configurations vary, while the measurements at zero (atmospheric) pressure are fairly consistent with each other (see e.g. Figure 1 in *Hashida et.al.* [1993]).

If the pressure independence of the tension-softening curve $\sigma_T(\delta)$ reported for Iidata granite for confining pressures up to 26.5 $MPa$ [*Hashida et.al.*, 1993] is observed in other materials, then PIFR dominates intrinsic rock strength starting from depth

$$H>\frac{\sigma_T}{\rho g}\sqrt{\frac{\Delta_T}{\Delta_T+\Delta_S}}, \tag{29}$$

where $\rho$ is rock density and $g$ is gravitational acceleration. According to the data of *Hashida et.al.* [1993], peak tensile strength of Iidata granite approximately equals 7 $MPa$. If we assume for estimate $\sigma_T=20\ MPa$ and $\rho=3\cdot10^3\ kg\,m^{-3}$, then even in the case of $\Delta_S=0$ (tip cavity is absent) PIFR prevails over intrinsic rock strength starting from depths as low as $0.7\,km$. In fact, PIFR dominates over intrinsic rock toughness at even shallower depths if the fluid doesn't penetrate up to the base of the process zone, as indicated by both experimental data [*Warpinski*, 1985; *Johnson and Cleary*, 1991] and theoretical considerations [*Rubin,* 1993]. If, for instance, $\Delta_S/\Delta_T\geq10^2$ (say, $\Delta_S>3m$ and $\Delta_T<3cm$), then crack propagation may be controlled by lithostatic pressure at depths less than $100m$.

Effects of confining pressure on fracture propagation are especially important at large depths. As a rough example, consider a crack at a depth of $100km$ ($p_\infty\sim3\ GPa$). Suppose that in one case the crack is completely filled with fluid ($\Delta_S=0$) and $\Delta_T$ is $\sim1mm$ (grain size) and in another case unwetted tip regions have a length of $\sim1m$. In the first case $K_Q\sim10^8 Pa\cdot m^{1/2}$, and in the second case $K_Q\sim3\cdot10^9 Pa\cdot m^{1/2}$, which is $10^2$–$3\cdot10^3$ times greater than the typical zero pressure values for rocks [*Atkinson and Meredith,* 1987]. In particular, this suggests that fracture resistance associated with dike emplacement at depth may not be negligible compared to the mechanical energy release rate and viscous dissipation due to a magma flow, which significantly complicates equations governing dike growth [*Spence and Turcotte*, 1985; *Lister and Kerr*, 1991].

## Conclusions

The purpose of this paper was to show that fracture criteria for hydro- or magmafractures at depth essentially depend on confining pressure. Exact solutions for the equilibrium overpressure, shape and volume of a fluid-driven crack were obtained in the frame of a Dugdale-Barenblatt (tension-softening) model. They describe, strictly speaking, a two-dimensional horizontal crack far from the body boundaries. It follows from the solutions that significant characteristics of equilibrium cracks, such as overpressure $\Delta p$, maximum opening $\delta_{max}$ and volume $V$, as well as apparent critical stress intensity factor $K_Q$, grow proportionally to the confining pressure $p_\infty$. From the mechanical point of view such an increase is equivalent to the increase of rock strength, i.e. to the existence of pressure induced fracture resistance (PIFR). This PIFR depends not only on pressure but also on the dimensions of the unwetted regions $\Delta_S$ near the crack tip, which possibly explains the difference in slopes of experimental $K_Q(p_\infty)$ curves obtained for the same material at different loading configurations. According to our estimates, PIFR exceeds intrinsic rock strength starting from quite shallow depth on the order of hundreds or even tens of meters.

**Acknowledgments.** We thank Allan Rubin, Steve Martel, Michael Deal and an anonymous reviewer for the useful comments that helped to clarify the subject of this paper.

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Ya. M. Khazan, Department of Tectonophysics, Institute of Geophysics, 32 Palladina St., Kiev, 252680, Ukraine (e-mail: root%igpnanu.kiev.ua@ua.net).

Yu. A. Fialko, Department of Geological and Geophysical Sciences, Princeton University, Princeton, NJ 08544 (e-mail: fialko@geo.princeton.edu).

Received: April 14, 1995 Revised: July 10, 1995
Accepted: July 13, 1995